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Figure.1 – Reproducing the test case.

Figure.1.a (LEFT): Reproduced test case of the Einstein ring from centred, single pixel source; lensed by equations for a planar, transparent, smooth mass distribution. The source image size is () pixels and lensing is completed with parameters: . The domain of reduced coordinates is kept to . The full ring is produced as expected, at the pre-calculated radius (for ) and a plot of the predicted ring is overlaid.

Figure.1.b (RIGHT): Same system scaled to a larger source image. It uses pixels with 1 pixel centre source and all same parameters. The code still clearly reproduces correct, expected behaviour, the ring is still full, is of approximately equal thickness throughout, and lies at the expected radius.

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Figure.2 – Generating and lensing a galaxy cluster.

Figure.2.a (LEFT): Generated cluster of 70 galaxies on a pixel image. The flux profile of all galaxies follows: . Galaxies have randomly generated: positions, angles to horizontal, RGB colours , decay constants in range pixels, minor axis in range pixels and major axis ensured to be larger than minor axis in range . For each galaxy, the parameter in the flux profile has been transformed using the standard 2D rotation matrix . The image has been seeded for reproducibility.

Figure.2.b (RIGHT): Lensed image of generated galaxies (from Figure.2.a) with used lensing parameters: and with domain of reduced coordinates giving reduced coordinate range of . The image clearly demonstrates the effect of far galaxies being stretched and pushed to further radii. It is also clear that the galaxy (blue) passing through the centre of the lens produces a nearly ideal Einstein ring with expected radius (). Galaxies near the centre can be viewed twice in the lensed image, as expected for objects positioned within the caustic of the lens.

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Figure.3 –jpg image lensed using a black hole lens

Figure.3.a (LEFT): Original jpg image of Messier 65 (credit: Maciej Tomasz Jarema). The jpg was cut to a square size of pixels.

Figure.3.b (RIGHT): Lensed jpg image with black hole lens . The used domain was . As expected, an Einstein ring is formed, of the expected size (here ). It is clear that the star (brown, left of centre) is seen twice in the lensed image, as would be expected for objects within the lens caustic. The same is true for the red feature (lower right of centre).

This lensing of a jpg has also been made into an animation, where it is moved into the lensing plane. There, Figure.3.b is the same as the last frame.

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Figure.4 – Parameter study, effect of lensing with varying on total luminosity.

An image ( pixels) of a circular source (radius pixels) has been lensed with varying ellipticities and the integrated bolometric luminosity for each has been found. Here, I define integrated bolometric luminosity as the sum of all RGB values, over all pixels. The fractional difference has then been found between the lensed image and the original and plotted for ellipticities in range . Inset plots demonstrate examples of seen images at ellipticities: and .

**PICTURE OF SOURCE, bigger labels**